Understanding Deep Learning (Still) Requires Rethinking Generalization

Alban MARIE [†]

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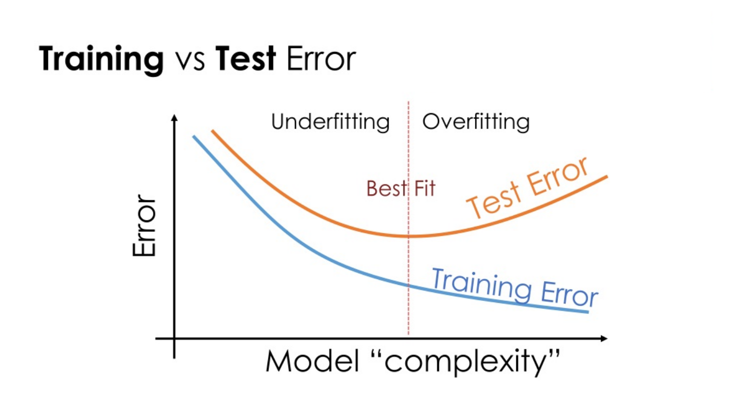
[†] Univ Rennes, INSA Rennes, CNRS, IETR - UMR6164, France

Why something is wrong with deep learning

Bias vs Variance



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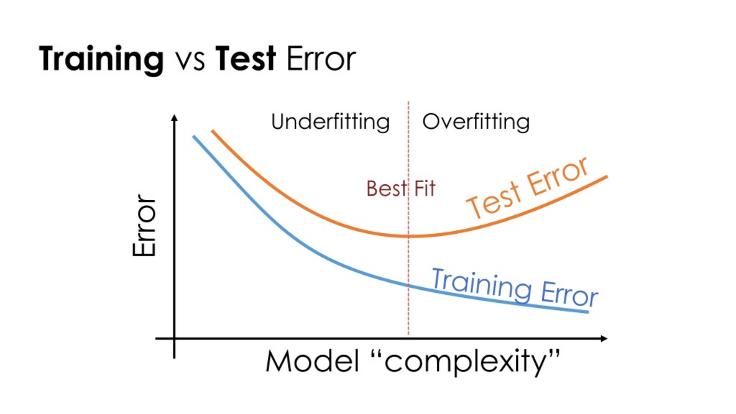


➜ Where is deep learning on the x-axis?

Underfitting vs Overfitting

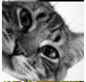
Where is deep learning on the x-axis?

Authors perform a simple experimental framework to propose an answer to this question.

Experimental framework

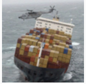
➜ True labels

Truck



Cat

Bird house



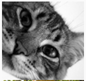
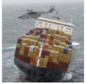
Container ship

Russian airplane probably

Dog

Experimental framework

➜ Random labels

	Truck	Russian airplane probably
	Cat	Cat
	Bird house	Container ship
	Container ship	Truck
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- ➜ Random pixels
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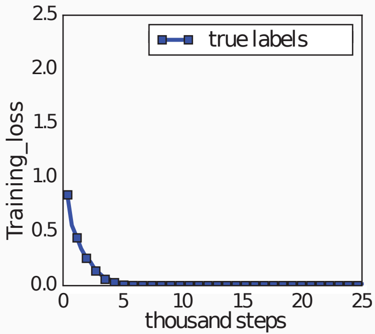
- ➜ Gaussian pixels
  - ➜ Independently for each pixel, draw a random value from gaussian distribution with mean and std from original dataset

Experimental framework

Ok cool but ...

... what's the point of these experiments?

Results



Training loss of true label experiment decaying with the training steps on CIFAR10

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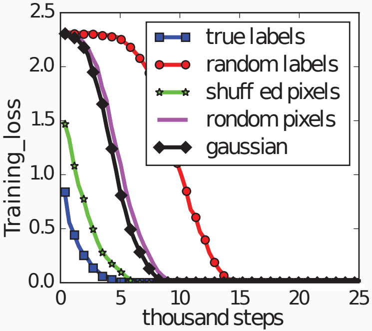

Training loss of various experiment settings decaying with the training steps on CIFAR10

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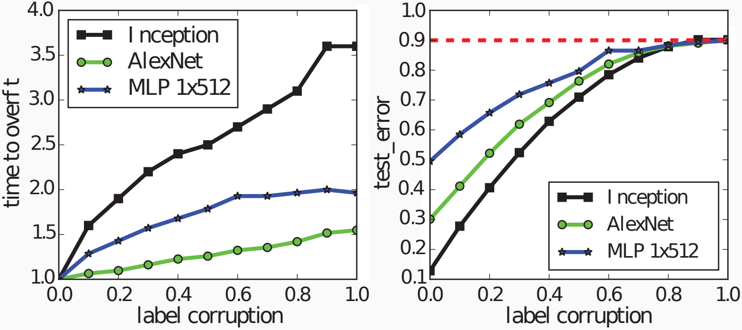

(left) Relative convergence time with different label corruption ratio and **(right)** test error (also the generalization error since training error is 0) under different label corruptions.

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- ➜ Optimization remains easy, whatever you aim to fit.
  - ➜ Easy even with random labels (the randomization breaks any relationship between the image and the label).

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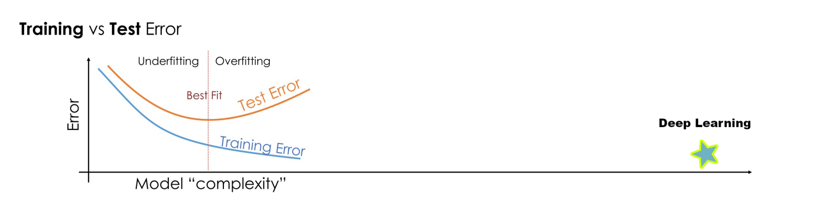

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  - Thus, very high overfitting
- At the same time, increasing model complexity allow to reduce test error (and thus generalization error since training error is 0)
  - Something is wrong

What about regularization?

Regularizers

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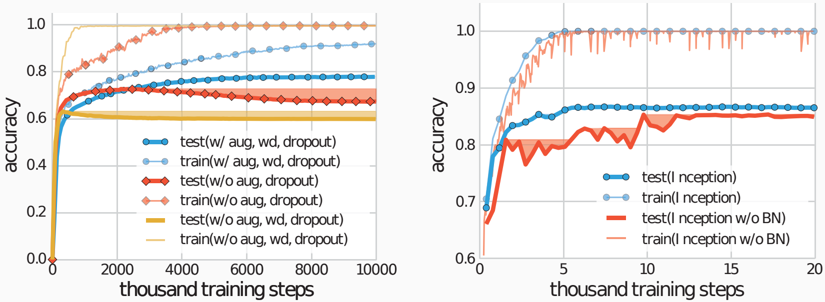
- ➜ NN architecture

Regularizers experiments



Regularizers impact on generalization for **(left)** Imagenet and **(right)** CIFAR10.
Data augmentation, **weight decay** and **batch normalization**
are referred as **aug**, **wd** and **BN**, respectively.

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  - ➜ it is unlikely that regularizers are the fundamental reason for generalization
- ➜ Implicit Regularization with NN architecture is more powerful to reduce generalization error

Bridge the gap with theory

VC Dimension - Statistical learning theory

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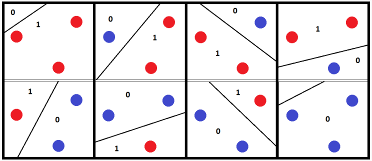

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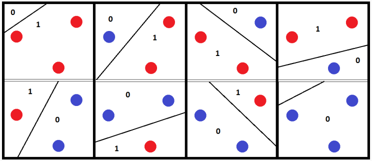

There is no set of 4 points that can be shattered by a line.

VC dim ≥ 3

VC dim < 4

VC Dimension - Statistical learning theory

Let N be the size of the dataset and D_{VC} the VC dimension of model f. With probability $1-\delta$:

$$err_{test} \leq err_{train} + \sqrt{\frac{1}{N}[D_{VC}(\log(\frac{2N}{D_{VC}})+1)-\log(\frac{\delta}{4})]}$$

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  - ➜ Thus, $D_{VC} \geq N$

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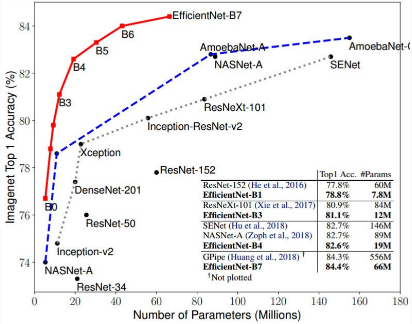
Theorem 1 *There exists a two-layer neural network with ReLU activations and $2N + d$ weights that can represent any function on a sample of size N in d dimensions.*

NN Finite Sample Expressivity

Dataset	N	d	$2N + d$
MNIST	70.000	$28^2 = 784$	140.784
CIFAR10	50.000	$3 \times 32^2 = 3.072$	103.072
ImageNet	$1.281.165$	$3 \times 224^2 = 150.528$	$1.431.693$



Number of parameters of ImageNet state-of-the-art models

NN Finite Sample Expressivity

This explains why NN manage to have 0 training error on random labels

They just have way too many parameters!

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- ➜ There is more in the paper, but I did not fully understand to present it

Thank you for listening!

Any questions?